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Chapter 7 - Arbitrage in FX Markets

Last Lecture
We went over effect of government on S_t
- FX rate regimes: Fixed, free float & mixed.
- CB sterilized (no effect on domestic Money Markets) and non-sterilized interventions.

This Lecture
Effect of arbitrage on S_t

Arbitrage
Definition: It involves *no risk* and *no capital of your own*. It is an activity that takes advantages of pricing mistakes in financial instruments in one or more markets. That is, arbitrage involves
(1) Pricing mistake
(2) No own capital
(3) No Risk

Note: The definition we used presents the ideal view of (riskless) arbitrage. "Arbitrage," in the real world, involves some risk (the lower, the closer to the pure definition of arbitrage). We will call this arbitrage *pseudo arbitrage*.

There are 3 types of arbitrage:
(1) Local (sets uniform rates across banks)
(2) Triangular (sets cross rates)
(3) Covered (sets forward rates)

1. Local Arbitrage (One good, one market)

It sets the price of one good in one market. Law of one price: the same good should trade for the same price in the same market.

Example: Suppose two banks have the following bid-ask FX quotes:

	Bank A		**Bank B**	
USD/GBP	1.50	1.51	1.53	1.55

Taking both quotes together, **Bank A** sells the GBP too low relative to **Bank B**'s prices. (Or, conversely, **Bank B** buys the GBP too high relative to **Bank A**'s prices). This is the pricing mistake!

Sketch of Local Arbitrage strategy:
(1) Borrow USD 1.51 (<= No own capital!)
(2) Buy a GBP from **Bank A** (at ask price $S^A_{t,ask}$ = USD 1.51)

(3) Sell GBP to **Bank B** (at bid price $S^{B}_{t,bid}$ = USD 1.53)
(4) Return USD 1.51 and make a π = USD .02 profit (1.31% per USD 1.51 borrowed)

Local Arbitrage Notes:
◊ All steps should be done simultaneously. Otherwise, there is risk! (Prices might change).
◊ **Bank A** and **Bank B** will notice a *book imbalance:*
- **Bank A**: all activity at the ask side (buy GBP orders –i.e., "GBP undervalued at $S^{A}_{t,ask}$")
- **Bank B**: all activity at the bid side (sell GBP orders –i.e., "GBP overvalued at $S^{B}_{t,bid}$").

Both banks will notice the imbalance and they will adjust the quotes. For example, **Bank A** will increase $S^{A}_{t,ask}$ and **Bank B** will reduce $S^{B}_{t,bid}$, say to 1.530 USD/GBP and 1.525 USD/GBP, respectively. ¶

2. Triangular Arbitrage (Two related goods, one market)

Triangular arbitrage is a process where two related goods set a third price. In the FX Market, triangular arbitrage sets FX cross rates. Cross rates are exchange rates that do not involve the USD. Most currencies are quoted against the USD. Thus, cross-rates are calculated from USD quotations –i.e., the most liquid quotes.

The cross-rates are calculated in such a way that arbitrageurs cannot take advantage of the quoted prices. Otherwise, triangular arbitrage strategies would be possible.

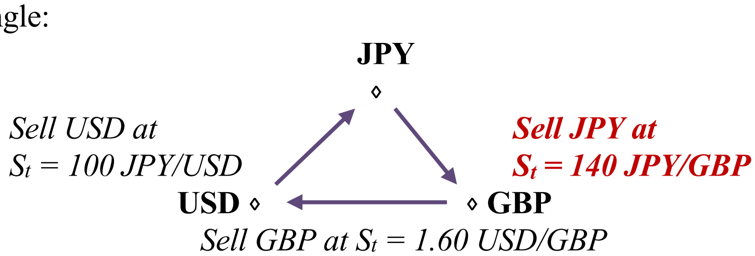
Example: Suppose Bank One gives the following quotes:
$S_{JPY/USD,t}$ = 100 JPY/USD
$S_{USD/GBP,t}$ = 1.60 USD/GBP
$S_{JPY/GBP,t}$ = 140 JPY/GBP
Take the first two quotes. Then, the implied (no-arbitrage) JPY/GBP quote should be:

$$S^{I}_{JPY/GBP,t} = S_{JPY/USD,t} \times S_{USD/GBP,t} = 160 \text{ JPY/GBP} > S_{JPY/GBP,t}$$

⇒ At $S_{JPY/GBP,t}$ = 140 JPY/GBP, Bank One *undervalues the GBP* against the JPY (with respect to the first two quotes). This is the pricing mistake!

Sketch of Triangular Arbitrage (Key: Buy undervalued GPB with the overvalued JPY):
(1) Borrow USD 1
(2) Sell USD/Buy JPY at $S_{JPY/USD,t}$ = 100 JPY/USD –i,e, sell the USD for JPY 100.
(3) Sell JPY/Buy GBP at $S_{JPY/GBP,t}$ = 140 JPY/GBP –i.e., sell JPY 100 for GBP 0.7143
(4) Sell GBP/Buy USD at $S_{USD/GBP,t}$ = 1.60 USD/GBP –i.e., sell the GPB 0.7143 for USD 1.1429
(5) Return loan, keep profits: π: USD 0.1429 (14.29% per USD borrowed).

The triangle:



Note: Bank One will notice a book imbalance (all the activity involves selling USD for JPY, selling JPY for GBP, selling GBP for USD.) and will adjust quotes. Say:

$S_{JPY/USD,t}\downarrow$ (say, $S_{JPY/USD,t}$ = 93 JPY/USD).
$S_{USD/GBP,t}\downarrow$ (say, $S_{USD/GBP,t}$ = 1.56 USD/GBP).
$S_{JPY/GBP,t}\uparrow$ (say, $S_{JPY/GBP,t}$ = 145 JPY/GBP).

There is convergence between $S^I_{JPY,GBP,t}$ & $S_{JPY,GBP,t}$:
$S^I_{JPY,GBP,t}\downarrow$ (= $S_{JPY,USD,t}\downarrow$ x $S_{USD/GBP,t}\downarrow$) $\Leftrightarrow$ $S_{JPY,GBP,t}\uparrow$ ¶

Again, all the steps in the triangular arbitrage strategy should be done at the same time. Otherwise, we'll be facing risk and what we are doing should be considered pseudo-arbitrage.

It does not matter which currency you borrow (USD, GBP, JPY) in step (1). As long as the strategy involves the step **Sell JPY/Buy GBP** (following the direction of the arrows in the triangle above!), you should get the same profit as a %.

3. Covered Interest Arbitrage (Four instruments -two goods per market-, two markets)

Open the third section of the WSJ: Brazilian bonds yield 10% and Japanese bonds 1%.

Q: Why wouldn't capital flow to Brazil from Japan?
A: FX risk: Once JPY are exchanged for BRL (Brazilian reals), there is no guarantee that the BRL will not depreciate against the JPY. The only way to avoid this FX risk is to be covered with a forward FX contract.

Intuition: Suppose we have the following data:
i_{JPY} = 1% for 1 year (T=1 year)
i_{BRL} = 10% for 1 year (T=1 year)
S_t = .025 BRL/JPY

We construct the following strategy, called *carry trade*, to "profit" from the interest rate differential:
Today, at time t=0, we do the following (1)-(3) transactions:

(1) Borrow JPY 1,000 at 1% for 1 year. (At T=1 year, we will need to repay JPY 1,010.)
(2) Convert to BRL at S_t = .025 BRL/JPY. Get BRL 25.
(3) Deposit BRL 25 at 10% for 1 year. (At T=1 year, we will receive BRL 27.50.)

Now, we *wait* 1 year. At time T=1 year, we do the final step:

(4) Exchange BRL 27.50 for JPY at S_{t+T}.

Problem with this carry trade: Today, we do not know $S_{t+T=1\text{-year}}$. Note:

- If S_{t+T} = .022 BRL/JPY, we will receive JPY 1250, for a profit of JPY 240.
- If S_{t+T} = .025 BRL/JPY, we will receive JPY 1100, for a profit of JPY 90.
- If S_{t+T} = .027 BRL/JPY, we will receive JPY 1019, for a profit of JPY 9.
- If S_{t+T} = .030 BRL/JPY, we will receive JPY 916, for a profit of JPY -74.

$\Rightarrow$ We are facing *FX risk*. That is, (1)-(4) is not an arbitrage strategy.

Now, at time t=0, we can use the FX forward market to insure a certain exchange rate for the JPY/BRL.
Suppose we get a quote of $F_{t,1-yr}$ = .026 JPY/BRL. At time t=0, we re-do step (4):

(4') Sell BRL forward at .026 JPY/BRL. (We will receive JPY 1058, for a sure profit of JPY 48.)

$\Rightarrow$ We are facing no FX risk. That is, **(1) - (4')** is an arbitrage strategy (*covered arbitrage*).

Now, instead of borrowing JPY 1,000, we will try to borrow JPY 1 billion (and make a JPY 48M profit) or more. Obviously, no bank will offer a .026 JPY/BRL forward contract!

7.3.1 Interest Rate Parity Theorem

Q: How do banks price FX forward contracts?
A: In such a way that arbitrageurs cannot take advantage of their quotes.

To price a forward contract, banks consider covered arbitrage strategies.

Review of Notation:
i_d = domestic nominal T days interest rate.
i_f = foreign nominal T days interest rate.
S_t = time t spot rate (direct quote, for example USD/GBP).
$F_{t,T}$ = forward rate for delivery at date T, at time t.
Note: In developed markets (like the USA), a*ll* interest rates are quoted on annualized basis. We will use annualized interest rates (The textbook is completely mistaken when it quotes periodic rates!!)

Now, consider the following (*covered*) strategy:

(1) At time 0, we borrow from a foreign bank 1 unit of a foreign currency (FC) for T days.
$\Rightarrow$ At time=T, We pay the foreign bank $(1 + i_f * T/360)$ units of the FC.

(2) At time 0, we exchange FC 1 at S_t $\Rightarrow$ for 1 unit of FC we get S_t.

(3) We deposit S_t in a domestic bank for T days.
$\Rightarrow$ At time T, we receive $S_t * (1 + i_d * T/360)$ (in DC).

(4) At time 0, we buy a T days forward contract to exchange domestic currency (DC) for FC at a $F_{t,T}$.
$\Rightarrow$ At time T, we exchange the DC $S_t * (1 + i_d * T/360)$ for FC, using $F_{t,T}$.
$\Rightarrow$ We get $S_t * (1 + i_d * T/360)/F_{t,T}$ units of FC.

If we do **(1)** - **(4)** simultaneously, this strategy faces No Risk. In equilibrium, no risk = no profits! This strategy will not be profitable if, at time T, what we receive in FC is less or equal to what we have to pay in FC. That is, arbitrage will ensure that

$$S_t\,(1 + i_d * T/360)/\,F_{t,T} = (1 + i_f * T/360). \qquad \text{(This is a No Arbitrage Condition!)}$$

Solving for $F_{t,T}$, we obtain the following expression:

$$F_{t,T} = S_t * \frac{(1+i_d * \frac{T}{360})}{(1+i_f * \frac{T}{360})} \qquad \text{(\textit{Interest Rate Parity Theorem} or \textit{IRPT})}$$

The IRP theory, also called *covered IRPT*, as presented above was first clearly exposed by John Maynard Keynes (1923).

It is common to use the following linear IRPT approximation:

$$F_{t,T} \approx S_t * [1 + (i_d - i_f) * \frac{T}{360}]$$

This linear approximation is quite accurate for small i_d & i_f (say, less than 10%).

Notes:
- ◊ Steps **(2)** and **(4)** simultaneously done produce a FX swap transaction! In this case, we buy the FC forward at $F_{t,T}$ and go sell the FC at S_t. We can think of ($F_{t,T}$ - S_t) as a profit from the FX swap.
- ◊ We get the same IRPT equation if we start the covered strategy by **(1)** borrowing DC at i_d; **(2)** exchanging DC for FC at S_t; **(3)** depositing the FC at i_f; and **(4)** selling the FC forward at $F_{t,T}$.

Example: IRPT at work.
Data:
S_t = 106 JPY/USD.
$i_{d=JPY}$ = .034.
$i_{f=USD}$ = .050.
$F_{t,T=1-yr}$ = ?
Using the IRPT formula:

$$F^{IRP}_{t,1-yr} = 106 \text{ JPY/USD} * \frac{(1+.034)}{(1+.05)} = \textbf{104.384 JPY/USD}.$$

Using the linear approximation:

$$F^{IRP}_{t,1-yr} = 106 \text{ JPY/USD} * (1 - .016) = 104.304 \text{ JPY/USD}.$$

⇒ The approximation error is less than 0.08%.

Note: If a bank sets $F^{A}_{t,1-yr}$ = 104.384 JPY/USD arbitrageurs cannot profit from the bank's quotes. ¶

Arbitrageurs can profit from any violation of IRPT.

Example 1: Violation of IRPT 1 - Undervaluation of forward FC (=USD, in this example).
Suppose IRPT is violated. Bank A offers: $F^{A}_{t,1-yr}$ **= 100 JPY/USD**.

⇒ $F^{A}_{t,1-yr}$ **= 100 JPY/USD** < $F^{IRP}_{t,1-yr}$ **= 104.384 JPY/USD** (a pricing mistake!): Bank A undervalues the forward USD against the JPY.

⇒ Take advantage of Bank A's undervaluation: **Buy USD forward** at $F^{A}_{t,1-yr}$.

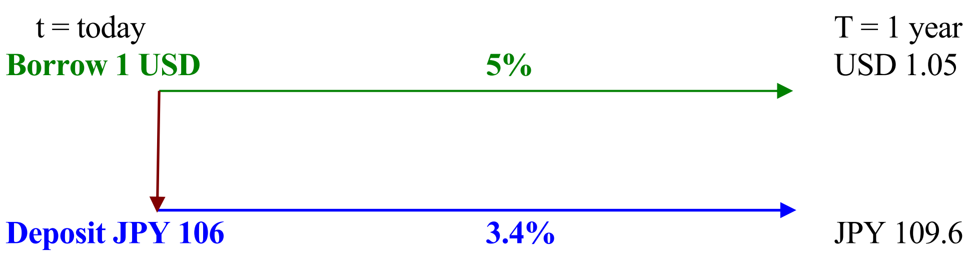
Sketch of a covered arbitrage strategy, summarized in Figure 7.1:

(1) Borrow USD 1 from a U.S. bank for one year at 5%.
(2) Convert USD to JPY at S_t = 106 JPY/USD
(3) Deposit the JPY in a Japanese bank at 3.4%.
(4) Cover. **Buy USD forward/Sell forward JPY** at $F^{A}_{t,1-yr}$ = 100 JPY/USD

Cash flows at time T=1 year,
(i) We get: JPY 106 * (1+.034)/(100 JPY/USD) = USD 1.096
(ii) We pay: USD 1 * (1+.05) = USD 1.05
Profit = Π = USD 1.096 – USD 1.05 = USD .046 (or 4.6% per USD borrowed)

After one year, the U.S. investor realizes a risk-free profit of USD. 046 per USD borrowed.

Figure 7.1: IRPT At Work



Note: Bank A will observe a lot of buying USD forward at $F^{A}_{t,1-yr}$ =100 JPY/USD. Bank A will quickly increase the quote until it converges to $F_{t,1-yr}$ = 104.38 JPY/USD. ¶

Example 2: Violation of IRPT 2 - Overvaluation of forward FC (=USD).
Now, suppose Bank X offers: $F^{X}_{t,1-yr}$ **= 110 JPY/USD**.
Then, $F^{X}_{t,1-yr} > F^{IRP}_{t,1-yr}$ (pricing mistake!) ⇒ The forward USD is overvalued against the JPY.
⇒ Take advantage of Bank X's overvaluation: **Sell USD forward**.

Sketch of a covered arbitrage strategy (shown in Graph 7.1):

(1) Borrow JPY 1 from for one year at 3.4%.
(2) Convert JPY to USD at S_t = 106 JPY/USD
(3) Deposit the USD at 5% for one year
(4) Cover. **Sell USD forward/Buy forward JPY** at $F^{X}_{t,1-yr}$ = 110 JPY/USD.

Cash flows at T=1 year:
(i) We get: USD 1/106 * (1 + .05) * (110 JPY/USD) = JPY 1.0896
(ii) We pay: JPY 1 * (1 + .034) = JPY 1.034
Π = JPY1.0896 – JPY 1.034 = JPY .0556 (or 5.56% per JPY borrowed)

Note: Arbitrage will ensure that Bank X's quote quickly converges to $F_{t,1-yr}$= 104.38 JPY/USD. ¶

• IRPT: Assumptions

Behind the covered arbitrage strategy -steps (1) to (4)-, we have implicitly assumed:
(1) Funding is available. Step (1) can be executed.
(2) Free capital mobility. No barriers to international capital flow –i.e., step (2) and, later, step (4) can be implemented.
(3) No default/country risk. Steps (3) and (4) are safe.
(4) Absence of significant frictions. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true. In general, the forward market is liquid for short maturities (up to 1 year). For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).

• IRPT and the Forward Premium

Recall the definition of forward premium, *p*:

$$p = \frac{F_{t,T} - S_t}{S_t} * \frac{360}{T}$$

Consider linearized IRPT. After some algebra, and letting T=360, we get:

$$p = \frac{F_{t,T} - S_t}{S_t} * \frac{360}{T} \approx (i_d - i_f).$$

p measures the annualized return from a long (short) position in the FX spot market and a short (long) position in the FX forward market. That is, it measures the return from an FX Swap transaction. We say if:
$p>0 \Rightarrow$ premium currency ("*the FC trades at a premium against the DC for delivery in T days*")
$p<0 \Rightarrow$ discount currency ("*the FC trades at a discount*")

Equilibrium: *p* exactly compensates $(i_d - i_f)$ → No arbitrage opportunities
→ No capital flows (because of pricing mistakes).

In Figure 7.2, the equilibrium points are along the 45º IRPT Line. Any pair (i_d -i_f, *p*) far from the IRPT Line represents an arbitrage opportunity.

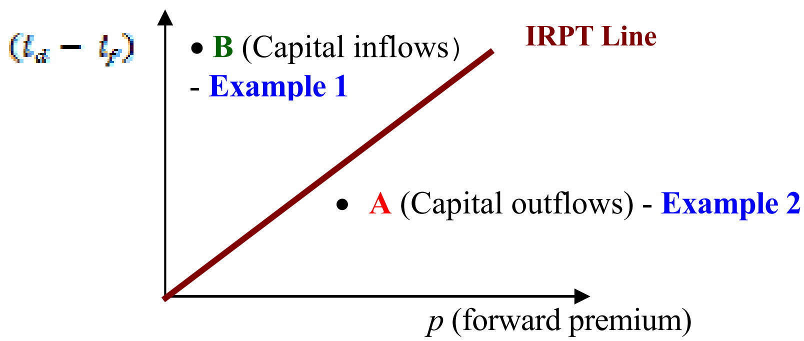
Example: Violations of IRPT and Capital Flows
B - Go back to **Example 1**
$p = [(F_{t,T} - S_t)/ S_t] * 360/T = [(100 – 106)/106] * 360/360 = - 0.0566 \Rightarrow$ USD trades at a discount.
$p = - 0.0566 < (i_d - i_f) = - 0.016 \quad \Rightarrow$ Arbitrage possible (pricing mistake!) $\Rightarrow$ capital flows!

Check Steps (1)-(3) in Example 1: Foreign (U.S.) capital flows to Japan (capital inflows to Japan).

A - Go back to **Example 2**
$p = [(F_{t,T} - S_t)/ S_t] * 360/T = [(110 – 106)/106] * 360/360 = 0.0377$ ⇒ USD trades at a premium.
$p = 0.0377 > (i_d - i_f) = - 0.016$ ⇒ Arbitrage possible (pricing mistake!) ⇒ capital flows!

Check Steps (1)-(3) in Example 2: Domestic (Japanese) capital flows to USA (capital outflows). ¶

Figure 7.2: IRPT Line



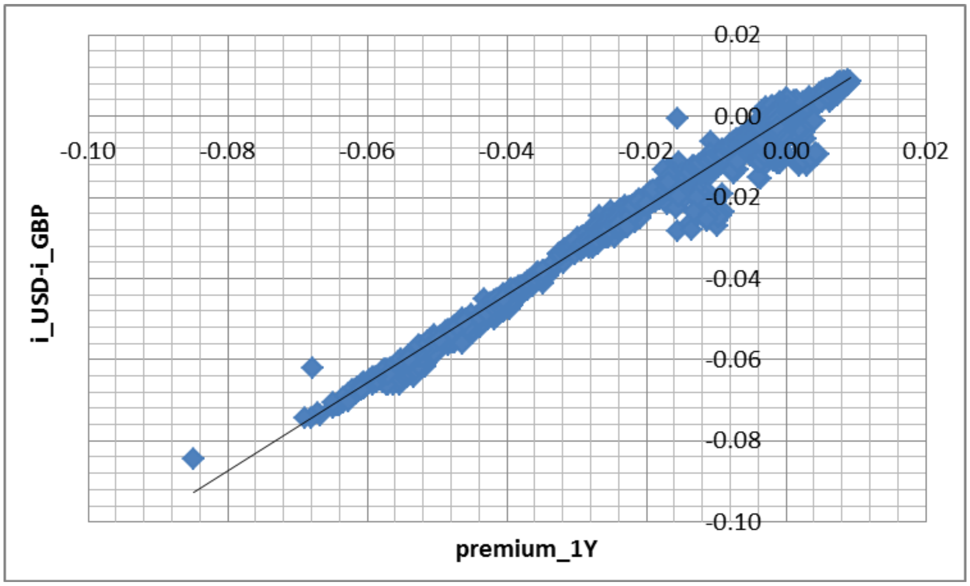
Consider a point under the IRPT line, say **A** (like in **Example 2**): $p > (i_d - i_f)$ (or $p + i_f > i_d$) → a long spot/short forward position has a higher yield than borrowing abroad at i_f and investing at home at i_d ⇒ Arbitrage is possible! (There is a pricing mistake).

Covered Arbitrage Strategy:
(1) Borrow DC at i_d for T days
(2) Convert DC to FC (the long position in FC)
(3) Deposit FC at i_f for T days
(4) Sell forward FC at $F_{t,T}$ (the short forward position in FC)

That is, today, at a point like **A**, domestic capital fly to the foreign country: What an investor pays in domestic interest rate, i_d, is more than compensated by the high forward premium, *p*, and the foreign interest rate, i_f.

• IRPT: Evidence
Starting from Frenkel and Levich (1975), there is a lot of evidence that supports IRPT. For example, Figure 7.3 plots the daily interest rate differential against the annualized forward premium. They plot very much along the 45° line. Moreover, the correlation coefficient between these two series is 0.995, highly correlated series!

Figure 7.3: IRPT Line – USD/GBP (daily, 1990-2015)



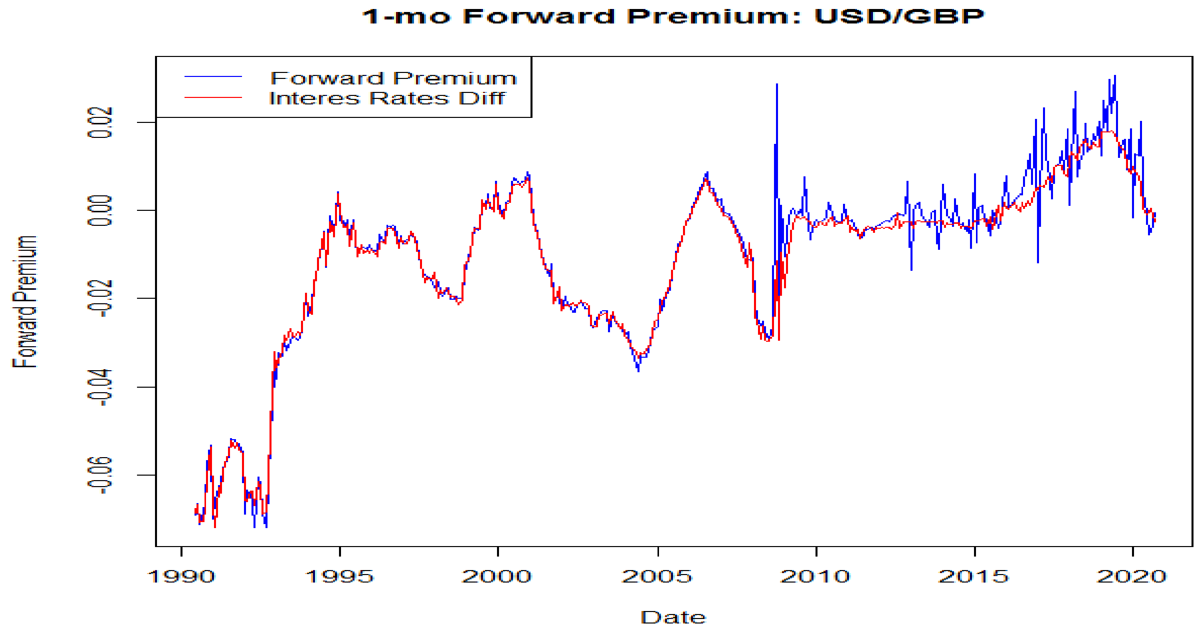
Using intra-daily data (10' intervals), Taylor (1989) also find strong support for IRPT. At the tick-by-tick data, Akram, Rice and Sarno (2008, 2009) show that there are short-lived (from 30 seconds up to 4 minutes) departures from IRP, with a potential profit range of 0.0002-0.0006 per unit. The short-lived nature and small profit range point out to a fairly efficient market, with the data close to the IRPT line.

There are situations, however, where we observe significant and more persistent deviations from the IRPT line. These situations are usually attributed to monetary policy, credit risk, funding conditions, risk aversion of investors, lack of capital mobility, default risk, country risk, and market microstructure effects.

For example, during the 2008-2009 financial crisis there were several violations of IRPT (in Graph 7.2, the point well over the line (-.0154,-.0005) is from May 2009). These violations are attributed to funding constraints –i.e., difficulties to do step (1): borrow. See Baba and Parker (2009) and Griffoli and Ranaldo (2011).

After the financial crisis, the behavior of the forward premium and interest differentials has changed. As seen in Figure 7.4, there was an almost perfect fit until 2008. Since 2008-2009, IRPT the fit is not that good. One explanation, the interest rates used are no longer "risk-free."

Figure 7.4: IRPT – Forward Premium & Interest Rate Differential (1990-2022)



Chapter 7 - Appendix – Taylor Series

Definition: Taylor Series

Suppose f is an infinitely often differentiable function on a set $\boldsymbol{D}$ and $c \in \boldsymbol{D}$. Then, the series

$$T_f(x, c) = \Sigma_n \, [\, f^{(n)}(c)/n!] \, (x - c)^n$$

is called the (formal) *Taylor series* of f centered at, or around, c.

Note: If c=0, the series is also called *MacLaurin Series.*

Taylor Series Theorem

Suppose $f \in C^{n+1}([a, b])$ -i.e., f is $(n+1)$-times continuously differentiable on $[a, b]$. Then, for $c \in [a,b]$ we have:

$$f(x) = T(x,c) + R = \frac{f(c)}{0!}(x-c)^0 \; + \; \frac{f^{/}(c)}{1!}(x-c)^1 \; + \frac{f^{//}(c)}{2!}(x-c)^2 \; + \ldots + \; \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + R$$

where $R_{n+1}(x) = \dfrac{1}{n!}\int f^{(n+1)}(p)(x-p)^n \, dp$

In particular, the $T_f(x, c)$ for an infinitely often differentiable function f converges to f iff the remainder $R_{(n+1)}(x)$ →converges to 0 as $n \to \infty$.

Example: 1st-order Taylor series expansion, around c=0, of $f(\text{x}) = \log(1 + \text{x}d)$, where d is a constant

$f(\text{x}) = \log(1+\text{x } d)$ $\qquad f(\text{x}_0=0) = 0$

$f'(\text{x}) = d/(1+\text{x } d)$ $\qquad f'(\text{x}_0=1) = d$

=> 1st-order Taylor's series formula (n=1):

$\log(1+x\,d) \approx T(x; c) = 0 + d\,(x - 0) = x\,d$ => if d=1, then $\log(1+x) \approx x$

Application: IRP Approximation

We start with IRP: $F_{t,T} = S_t * \frac{(1 + i_d * \frac{T}{360})}{(1 + i_f * \frac{T}{360})}$

Now, take logs: $\log(F_{t,T}) - \log(S_t) + \log(1 + i_d * \frac{T}{360}) - \log(1 + i_f * \frac{T}{360})$

After simple algebra: $\log(F_{t,T}) - \log(S_t) = \log(1 + i_d * \frac{T}{360}) - \log(1 + i_f * \frac{T}{360})$

Recall that log changes can approximate percentage changes: $\log(F_{t,T}) - \log(S_t) \approx \frac{F_{t,T} - S_t}{S_t}$

Then, using the approximation for $\log(1 + xd) \approx x\,d$, we get $p = \frac{F_{t,T} - S_t}{S_t} \approx (i_d * \frac{T}{360} \quad i_f * \frac{T}{360})$

Solving for $F_{t,T}$, gets the linearized approximation to IRP.

CHAPTER 7 - BONUS COVERAGE: IRPT with Bid-Ask Spreads

Exchange rates and interest rates are quoted with bid-ask spreads.

Consider a trader in the interbank market:

She will have to buy or borrow at the other party's ask price.
She will sell or lend at the bid price.

There are two roads to take for arbitrageurs: borrow domestic currency or borrow foreign currency.

• Bid's Bound: Borrow Domestic Currency

At time $t = 0$, an arbitrager simultaneously would do:
(1) A trader borrows DC 1 at time $t = 0$ at $i_{ask,d}$
(2) Using the borrowed DC 1, she buys FC spot at $S_{ask,t}$, getting (1/$S_{ask,t}$)
(3) She deposits the FC at the foreign interest rate, $i_{bid,f}$.
(4) She sells the FC forward for T days at $F_{bid,t,T}$.

Note: The arbitrager always gets the "worst" part of the bid-ask spread.

Cash flows (in DC) at time T:
- Arbitrager repays: $1 + i_{ask,d} * T/360$.

- Arbitrager gets: $(1/S_{ask,t})\ (1 + i_{bid,f} * T/360) * F_{bid,t,T}$.

In equilibrium, this strategy should yield no profit. That is,

$$(1/S_{ask,t})\ (1 + i_{bid,f} * T/360) * F_{bid,t,T} \leq (1 + i_{ask,d} * T/360).$$

Solving for $F_{bid,t,T}$:

$$F_{bid,t,T} \leq S_{ask,t} * \frac{\left(1 + i_{ask,d} * \frac{T}{360}\right)}{\left(1 + i_{bid,f} * \frac{T}{360}\right)} = \mathbf{U}_{\mathbf{bid}}.$$

• <u>Ask's Bound: Borrow Foreign Currency</u>

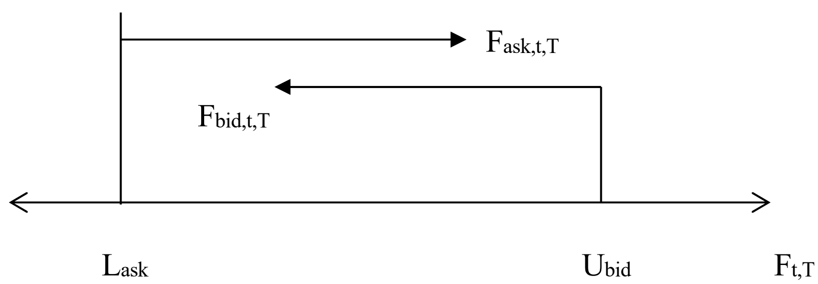
(1) The trader borrows FC 1 at time $t = 0$ at $i_{ask,f}$.
(2) Using the borrowed FC 1, she sells the FC spot for $S_{bid,t}$ units of DC.
(3) She deposits the DC at the domestic interest rate, $i_{bid,d}$.
(4) She buys the FC forward for T days at $F_{ask,t,T}$

Following a similar procedure as the one detailed above, we get:

$$F_{ask,t,T} \geq S_{bid,t} * \frac{\left(1 + i_{bid,d} * \frac{T}{360}\right)}{\left(1 + i_{ask,f} * \frac{T}{360}\right)} = \mathbf{L}_{\mathbf{ask}}.$$

Together with $F_{ask,t,T} > F_{bid,t,T}$, these bounds are used by traders to establish arbitrage-free quotes. Figure 7.4 illustrates the bounds.

<u>Figure 7.4</u>: Trading bounds for the Forward bid and the Forward ask.



Example: IRPT bounds at work.
Data: S_t = 1.6540-1.6620 USD/GBP
i_{USD} = 7¼-½,
i_{GBP} = 8 1/8–3/8,
$F_{t,one\text{-}year}$ = 1.6400-1.6450 USD/GBP.

Check if there is an arbitrage opportunity (we need to check the bid's bound and ask's bound).

i) Bid's bound covered arbitrage strategy:
1) Borrow USD 1 at 7.50% for 1 year => we will repay USD 1.07500 at T=1 year
2) Convert to GBP => we get GBP 1/1.6620 = GBP 0.6617

3) Deposit GBP 0.6017 at 8.25%
4) Sell GBP forward at 1.64 USD/GBP => we get (1/1.6620) x (1 + .08125)x1.64 = USD 1.06694
=> No arbitrage opportunity. For each USD we borrow, we lose USD .00806.

ii) Ask's bound covered arbitrage strategy:
1) Borrow GBP 1 at 8.375% for 1 year => we will repay GBP 1.08375 at T=1 year
2) Convert to USD => we get USD 1.6540
3) Deposit USD 1.6540 at 7.250%
4) Buy GBP forward at 1.645 USD/GBP => we get 1.6540x(1 + .07250)x(1/1.6450) = GBP 1.07837
=> No arbitrage opportunity. For each GBP we borrow, we lose GBP 0.0054.

Note: The bid-ask forward quote is consistent with no arbitrage. That is, the forward quote is within the IRPT bounds. Check:

$\mathbf{U_{bid}} = S_{ask,t} * \frac{(1+i_{ask,d})}{(1+i_{bid,f})}$ = 1.6620 USD/GBP * $\frac{(1.075)}{(1.08125)}$ = 1.6524 USD/GBP

$\Rightarrow \mathbf{U_{bid}} \geq F_{bid,t,T}$, = 1.6400 USD/GBP.

$\mathbf{L_{ask}} = S_{bid,t} * \frac{(1+i_{bid,d})}{(1+i_{ask,f})}$ = 1.6540 USD/GBP * $\frac{(1.0275)}{(1.08375)}$ = 1.6368 USD/GBP

$\Rightarrow \mathbf{L_{ask}} \leq F_{ask,t,T}$ = 1.6450 USD/GBP. ¶

CHAPTER 7 – BRIEF ASSESMENT

1. Assume the following information:

$S_{t.USD/AUD}$ = .8 USD/AUD
$S_{t,USD/GBP}$ = 1.40 USD/GBP
$S_{t,AUD/GBP}$ = 1.80 AUD/GBP

Is triangular arbitrage possible? If so, explain the steps that would reflect triangular arbitrage, and compute the profit from this strategy (expressed as a % per unit borrowed). Explain how market forces move to eliminate triangular arbitrage's profits.

2. Difficult. Let's complicate triangular arbitrage, by introducing bid-ask spreads. Assume the following information:

$S_{t,USD/AUD}$ = .81-.82 USD/AUD
$S_{t,USD/GBP}$ = 1.40-1.42 USD/GBP

Calculate an arbitrage-free cross rate (AUD/GBP) quote (with bid-ask spread).

3. Assume the following information:

S_t = 1.10 USD/EUR
i_{EUR} = 1.50%
i_{USD} = 2.75%
T = 180 days

(A) Determine the arbitrage-free 180-day forward rate (use IRP).
(B) Suppose Bank Q offers $F^Q_{t,180}$ = 1.12 USD/EUR. Given this information, is covered interest arbitrage possible? Design a covered arbitrage strategy and calculate its profits.
(C) Suppose Bank P offers $F^P_{t,180}$ = 1.08 USD/EUR. Given this information, is covered interest arbitrage possible? Design a covered arbitrage strategy and calculate its profits.

4. Assume the following information:

S_t = 1.40 USD/GBP
$F^Q_{t,270}$ = 1.42 USD/GBP
i_{GBP} = 2.50%
i_{USD} = 2.75%
T = 180 days

Calculate *p* (the forward premium) and the interest rate differential. What kind of capital flows the U.K. economy will experience?